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ABSTRACT

An activity for secondary schools is presented and discussed which may be explored with a programmable calculator. The activity is non-standard and could not be easily explored without the use of a programmable calculator. Related activities are also discussed. Flow charts and programs for different programmable calculators are presented. (MP)

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$x = \frac{1}{x} + 1$, A Programmable Calculator Activity[1]

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Programmable calculators as well as computers are ideal problem solving tools for secondary students and ideal teaching aids for secondary teachers. In fact, programmable calculators are often better devices for teaching and learning purposes than computers for several reasons including:

1. they now cost so little (a TI 57 is less than $50 and a HP 33 E less than $90) that individuals and schools are increasingly able to afford them;

2. they are small, hand-held and can be operated from rechargeable battery packs for complete portability;

3. they provide instant access, there is no waiting for that one, always busy, terminal; and

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[1] The authors are presenting variations of this activity in several journals in order to invite reader reaction and correspondence about non-standard problems that can be solved with programmable calculators.

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4. they solve many of the same kinds of problems handled by computers.

The purpose of this article is to share one problem (activity) which you and your students can explore with a calculator. Importantly, the activity has two key features. First, it is non-standard. That is, it is an activity that would not be easily explored without the use of a programmable calculator. Second, the activity can be used to motivate a discussion of some interesting mathematics as will be noted following the statement of the activity.

The Activity

I. a. Start with any number;

b. Take the reciprocal;

c. Add 1; and

d. Repeat from step b.

Figures 1 and 2 present flow charts and programs for the TI 57 and HP 33 E programmable calculators, respectively. These machines are chosen as they represent the least expensive models of the two major domestic manufacturers and will likely, therefore, be more widely available than other models.

Some relevant questions which might be asked about this particular activity include:

A. For a given starting number, say 5, what happens (to the value in the display) as the number of times through the loop increases?

B. For different positive integer starting numbers what happens (to the value in the display)?

C. For a starting number of 0, what happens (to the value in the display)? Are there other integers which give the same result?

D. For different negative integer starting numbers what happens

(to the value in the display)?

E. For different non-integer starting numbers what happens (to the value in the display)?

F. For what starting number(s) does the value in the display remain constant?

Analysis

If you explore the activity you will find that for virtually any starting number the value in the display tends to the value 1.618034 (on the TI 57) or the value 1.618033989 (on the HP 33 E) as a limit. We like this type of activity because it motivates the need for a technique to convince us that the limit is really what it appears to be, namely, the golden ratio.

Interestingly, it is easy to show that the limit of the sequence of numbers in the display is really the golden ratio. Think of the limiting value as x. Then x has the property that its reciprocal plus one is itself. That is,

$$(1)\quad x = \frac{1}{x} + 1$$

This equation leads to the quadratic $x^2 - x - 1 = 0$ whose roots are $\frac{1 \pm \sqrt{5}}{2}$.

The positive root $\frac{1 + \sqrt{5}}{2}$ gives the values 1.618034 and 1.618033989 to six and nine places, respectively, and is the famous golden ratio.

Related Activities

How does one create activities of this type? Since the activity leads to equation (1) which can be rearranged as a quadratice equation start with a quadratice equation and reverse the process. For example start with the quadratic $(x-3)(x+2) = 0$ and rewrite it as $x^2 = x + 6$ and then either take the square root obtaining the equation

$$(2)\quad x = \sqrt{x + 6}$$

or divide by x obtaining the equation

$$(3)\quad x = 1 + \frac{6}{x}$$

Each of these forms indicates a related activity which you and your students can explore.

Equation (2) suggest the activity:

II. a. Start with any number;

b. Add 6;

c. Take the square root; and

d. Repeat from step b.

Similarly, equation (3) suggests the additional activity:

III. a. Start with any number;

b. Divide it into 6;

c. Add 1; and

d. Repeat from step b.

What happens when you explore these activities? ... similar activities derived from other quadratic equations? ... activities derived from other equations such as cubics?

Conclusion

Hand-held programmable calculators offer an inexpensive alternative to computers as a tool for solving a variety of non-standard problems. Much study, research and classroom experimentation is needed to find suitable problems (activities) and ways to use these powerful calculators to enhance the teaching and learning of mathematics at the pre-college level. The authors would enjoy corresponding with persons who are using or know of non-standard problems which can be solved at the secondary level with programmable calculators, particularly the TI 57 and HP 33 E (or their predecessors the TI 56 and HP 25). Please direct any correspondence to Professor Spikell at 20 Pinebrook Road, Wayland, MA 01778.

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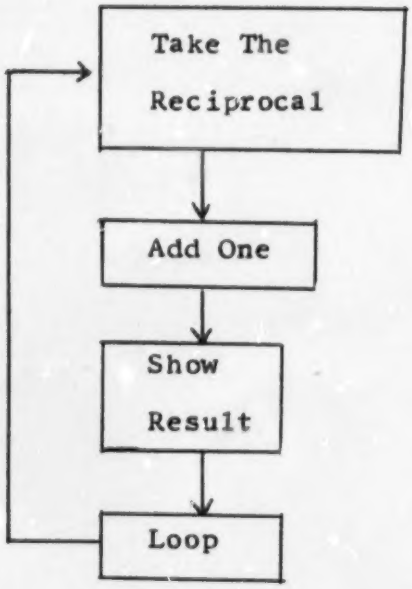


00 1/x

01 +
02 1
03 =

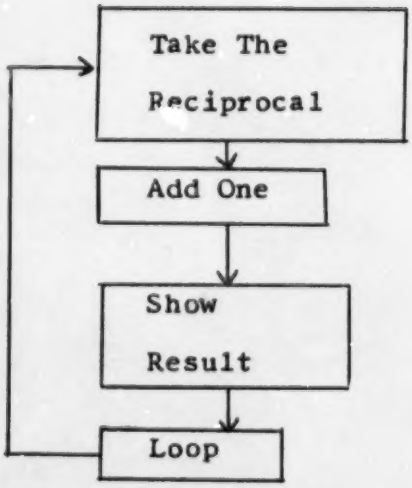
04 2nd Pause

05 RST

initialize: RST, enter any number, R/S

Figure 1

00 (R/S)



01 g 1/x

02 1
03 +

04 f PAUSE

05 GTO 01

initialize: f PRGM, enter any number, R/S

Figure 2